A Bibliography on Control of Piecewise-Affine Hybrid Systems on Polytopes

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A bibliography is provided on control of piecewise-affine hybrid systems on polytopes and closely related topics. This document is made for the participants of a Ph.D. School on Cyber-Physical System held in Verona, Italy during several days of September 2017.

1 Introduction

The reader finds in this document a structured list of references on control of piecewise-affine hybrid systems on polytopes and related topics.

This document is produced for the participants of the Ph.D. School on Cyber-Physical Systems scheduled for 11-15 September 2017 at the Department of Computer Science of the University of Verona in Verona, Italy.

The main references for the lectures of the author of this documents are [80, 76, 48]. Of the three references mentioned above, the first one deals only with affine systems on polytopes, the second with piecewise-affine hybrid systems on simplices, and the last one with output feedback of piecewise-affine hybrid systems on polytopes. The acronym PAHSP stands for piecewise-affine hybrid systems on polytopes. The results were developed at the research institute CWI in Amsterdam, The Netherlands, by the Research Group Control and System Theory, in particular by Pieter Collins, Luc Habets, and Jan H. van Schuppen.

Others have developed the approach in new directions.

The reader should note that almost all references are for continuous-time hybrid systems. When discrete-time systems are referred to this is explicitly mentioned.

2 List of References on Control of Piecewise-Affine Hybrid Systems on Polytopes (PAHSP)

Papers on control of PAHSP by the CWI Research Group Control and System Theory The initial paper on control of a piecewise-affine hybrid system on polytopes in which the control-to-facet approach was indirectly formulated is [147]. The main papers of the group are [80, 76, 48]. The complete list of papers follows, [147, 78, 79, 82, 81, 80, 83, 42, 76, 43, 47, 48].

Paper on abstraction of biochemical reaction systems, [46].

Papers on reach control by the Control Research Group of the University of Toronto The main investigator of the group is M.E. Broucke. [126, 127, 108, 109, 26, 25, 27, 110, 134, 84, 85, 29, 3, 88, 87, 135, 28, 149, 89, 86, 115, 102, 148]

Control engineering problems for a piecewise-affine hybrid system [5, 10, 94, 149, 148].

Papers on control of PAHSP by other research groups [94, 154].

Papers on Control and Verification by a research group of Boston University The main investigator of this group is C. Belta. [11, 7, 8, 10, 9, 77, 75].

Papers by the Control Research Group of Aalborg University The main investigator of the group is R. Wisniewski. [18, 103, 136, 106]. Control to facet problems for polynomial systems, [136].

3 Control of Hybrid Systems – General References

Papers on piecewise-linear hybrid systems The following papers by E.D. Sontag focus attention on the class of piecewise-affine hybrid system on polytopes as a class of hybrid systems for which theoretical progress and effective computations were possible. [138, 140, 141].

Related papers on control of continuous-time hybrid systems [2, 55, 111].

Papers on control of linear or affine systems [15, 16, 17, 32, 33, 74, 73].

Stability of piecewise-affine hybrid systems [].

Papers on discrete-time piecewise-affine hybrid systems by an ETH research group The main investigators were A. Bemporad and M. Morari. The restriction here is to discrete-time systems rather than the continuous-time systems investigated in most other references of this list. [12, 13]

Control of discrete-time linear systems with input constraints [51].

Decidability or undecidability of control of hybrid systems [92, 137, 140].

4 Observability and Observers of Piecewise-Affine Hybrid Systems

Observers of piecewise-affine hybrid systems The main investigators are Elena De Santis and Marika Di Benedetto. A tutorial paper, [132].

Papers on observability by the CWI Research Group Control and System Theory [45, 44].

5 Invariant and Controlled-Invariant Subsets of the State Space

Invariant Subspaces For invariant subspace of ordinary linear systems see the classical book, [152, 153].

Invariant Polyhedral Sets [90, 145].

Invariant Sets – General Case [19, 65, 96, 116, 118, 155].

Controlled-Invariant Subspaces For controlled-invariant sets of ordinary linear systems see the classical book, [152, 153].

Controlled-Invariant Subsets Papers by P.O. Gutman etal, [74].
Papers by F. Blanchini etal., [19, 20, 21, 23].
Papers by E. De Santis etal., [129, 130, 131].
Papers by others, [14, 69, 70, 91, 104, 146, 156].

6 Papers on Abstraction of Piecewise-Affine Hybrid Systems

Papers by an INRIA Research Group The main investigator is H. de Jong. [53, 54, 52, 6, 125].

Papers by a research group of Boston University The main investigator is C. Belta. [7, 75].

Papers by a Research Group of Masaryk University Brno The main investigator is David Safranek who teaches a course on bioinformatics. [46].

Papers on abstraction of piecewise-affine systems [4].

7 Realization Theory of Hybrid Systems

Realization theory Realization of hybrid systems is discussed. Realization of hybrid systems on polytopes is not discussed in the following papers. [121, 120, 122, 119].

8 Piecewise-linear systems

Publications of UCB research group In able to investigate the class of nonlinear systems in particular models of circuits, the class of piecewise-linear systems was considered at UC Berkeley. The main investigator was E.S. Kuh. [60, 61]. An even earlier paper is [99]. A related paper is [97].

Publications of another UCB research group Subsequently the class of piecewise-linear systems was investigated more deeply at UC Berkeley by Leon Chua and co-workers. The main investigator was L.O. Chua. [34, 39, 35, 37, 36, 38, 100, 101].

9 Background References in Control Theory and in Control Engineering

Books on control engineering Control engineering books include [56, 59].

Books on control theory An elementary introduction to control theory for researchers from outside control, is the book [1].

Books on linear systems are [31, 98]. A book written for readers both of control and of computer science is the book, [117]. For mathematical control theory see the book, [142]

Books on hybrid systems and switched systems [67, 107, 112, 144].

Books and articles on discontinuous systems and their control [40, 49].

10 Convexity and Polytopes

Convexity [124, 151, 22].

Polytopes Books on polytopes are [24, 68, 71, 72, 114, 157]
Articles on polytopes include, [30, 66, 133].

Triangulations of polytopes [50, 58, 105].

Projections of polytopes [123].

Piecewise-linear algebra [139].

Papers on Packages for Computations with Polytopes [62, 64, 63, 143].

11 Background References in Mathematics

Differential equations [41, 93, 150]

Filippov solution of a discontinuous ordinary differential equation [57].

Piecewise-linear topology [95, 113, 128].

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